M2 subject "ODE characterisation of entropic optimal transport"

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November 20, 2023

A natural way to solve numerically the optimal transport problem is by means of the well known entropic regularization: that is, given 2 probability measures μ_1 and μ_2 and a cost function $c : X_1 \times X_2 \to \mathbb{R}_+$ the minimization problem is defined on the set $\Pi(\mu_1, \mu_2)$ of coupling probability measures $\gamma \in \mathcal{P}(X_1 \times X_2)$ with marginals (μ_1, μ_2) as

$$\boxed{\mathsf{OT}^c_\varepsilon(\mu_1, \mu_2) := \inf_{\gamma \in \Pi(\mu_1,\mu_2)} \int_{X_1 \times X_2} c \mathrm{d}\gamma + \varepsilon \mathrm{Ent}(\gamma|\mu_1 \otimes \mu_2),} \tag{1}$$

where $\mathrm{Ent}(\gamma|\mu_1 \otimes \mu_2)$ is the Boltzmann-Shannon relative entropy (or Kullback-Leibler divergence) w.r.t. the product measure $\mu_1 \otimes \mu_2$, defined as $\int_{X_1 \times X_2} \rho \log \rho \, \mathrm{d}(\mu_1 \otimes \mu_2)$ if $\gamma = \rho \, \mu_1 \otimes \mu_2$ (and $+\infty$ otherwise). Notice that when it comes to solve the regularized problem (1) by using the alternating Kullback-Leibler projections on the 2 marginals constraints, by the algebraic properties of the entropy such an algorithm corresponds to the celebrated Sinkhorn's algorithm which is nowadays widely used to numerically solve optimal transport and related variational problems (see for instance [PC19]). Now, since $\mathsf{OT}^c_\varepsilon$ can be seen as perturbation of OT^c it is natural to study the behaviour as ε vanishes. The main purpose of this internship is to better understand this behaviour by exploiting an ODE characterisation of (1) as the one developed in [CM94, Wee, Del] for discrete and semi-discrete cases. Moreover, we also plan to tackle some variant of the entropic optimal transport such as the multi-marginal one [NP23], the barycenter, martingale OT, etc.

The M2 intern will be based at LMO (Laboratoire de mathématiques d'Orsay), Paris-Saclay university and will be supervised by L. Nenna (LMO) and P. Pegon (U. Paris-Dauphine). Moreover, he/she will benefit of the high quality scientific environment of LMO and interact with the members of a ParMa Inria team based at LMO as well as the Mokaplan team at Inria Paris and the ANR GOTA project members.

Please send a CV and transcripts.

References

[CM94] R. Cominetti and J. San Martín. Asymptotic analysis of the exponential penalty trajectory in linear programming. *Mathematical Programming*, 67(1):169–187, October 1994.

[Del] Alex Delalande. Nearly Tight Convergence Bounds for Semi-discrete Entropic Optimal Transport.

[NP23] Luca Nenna and Paul Pegon. Convergence rate of entropy-regularized multi-marginal optimal transport costs. *arXiv preprint arXiv:2307.03023*, 2023.

[PC19] Gabriel Peyré and Marco Cuturi. Computational Optimal Transport: With Applications to Data Science. *Foundations and Trends in Machine Learning*, 11(5-6):355–607, February 2019. Publisher: Now Publishers, Inc.

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